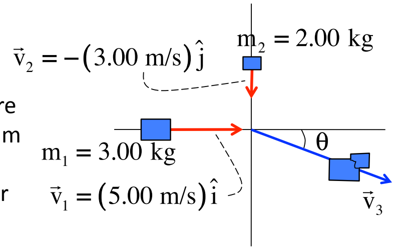
Problem 9.29

This is another classic *conservation of momentum* problem. All the forces acting are internal to the system, so the momentum sum in the x-direction must stay the same even beyond the collision, and the same is true for the y-direction. Using that, we can write:



$$\sum p_{1,x} + \sum F_{external,x}\Delta t_{throughCollision} = \sum p_{2,x}$$

$$\Rightarrow \quad m_1 v_1 = (m_1 + m_2) v_3 \cos\theta$$

$$\Rightarrow \quad v_3 \cos\theta = \frac{m_1 v_1}{(m_1 + m_2)}$$

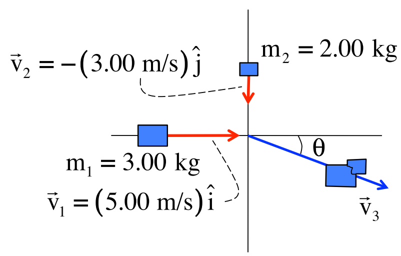
and with velocity signs unembedded to make the "v" terms magnitudes:

$$\sum p_{1,y} + \sum F_{external,y}\Delta t_{throughCollision} = \sum p_{2,y}$$

$$\Rightarrow \quad -m_2 v_2 = -(m_1 + m_2) v_3 \sin\theta$$

$$\Rightarrow \quad v_3 \sin\theta = \frac{m_2 v_2}{(m_1 + m_2)} \qquad \text{(note that the velocities are magnitudes)}$$

The problem with all of this falderah is that we end up with three unknowns: v_3, $\cos\theta$ and $\sin\theta$. To solve this, we have to be tricky. Specifically, notice that by *dividing* the two *conservation of momentum* relationships, we get:



$$\frac{\cancel{v_3}\sin\theta}{\cancel{v_3}\cos\theta} = \frac{\dfrac{m_2 v_2}{\cancel{(m_1 + m_2)}}}{\dfrac{m_1 v_1}{\cancel{(m_1 + m_2)}}}$$

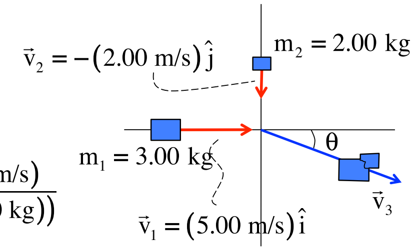
$$\Rightarrow \quad \frac{\sin\theta}{\cos\theta} = \tan\theta = \frac{m_2 v_2}{m_1 v_1}$$

$$\Rightarrow \quad \theta = \tan^{-1}\left(\frac{m_2 v_2}{m_1 v_1}\right)$$

$$= \tan^{-1}\left(\frac{(2.00 \text{ kg})(3.00 \text{ m/s})}{(3.00 \text{ kg})(5.00 \text{ m/s})}\right)$$

$$= 21.8^\circ$$

With the angle, we can write:



$$v_3 \cos\theta = \frac{m_1 v_1}{(m_1 + m_2)}$$

$$\Rightarrow \quad v_3 \cos 21.8^\circ = \frac{(3.00 \text{ kg})(5.00 \text{ m/s})}{((3.00 \text{ kg}) + (2.00 \text{ kg}))}$$

$$\Rightarrow \quad v_3 = 3.23 \text{ m/s}$$

(You'd have gotten the same result if you'd used the other equation.)

With that, finally, we can write:

$$\vec{v}_3 = (3.23 \text{ m/s}) \measuredangle (-21.8^\circ)$$

$$= ((3.23 \text{ m/s})\cos 21.8^\circ)\hat{i} - ((3.23 \text{ m/s})\sin 21.8^\circ)\hat{j}$$

$$= (3.00 \text{ m/s})\hat{i} - (1.20 \text{ m/s})\hat{j}$$